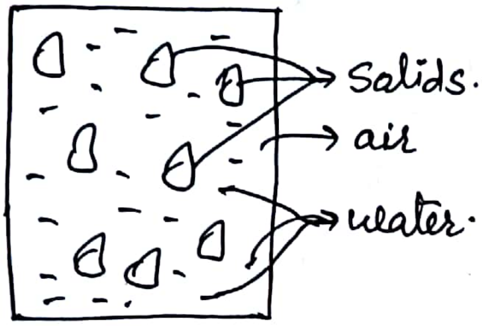
CHAPTER-2

(Physical and Index Properties of soil.)

1) Soil as a three phase system :-

Soil mass consists of solid particles, water and air. This arrangment is known as three phase diagram or Block diagram.



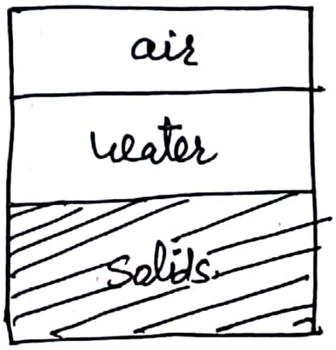
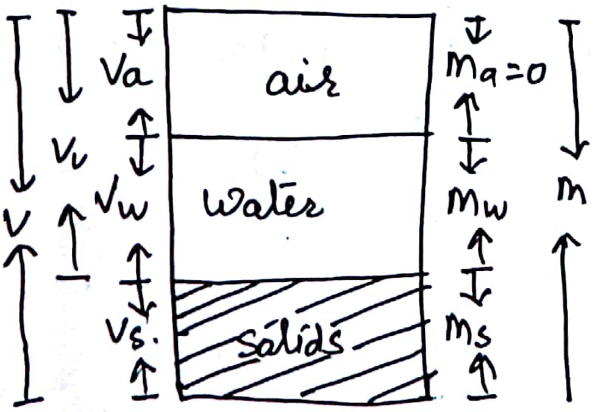
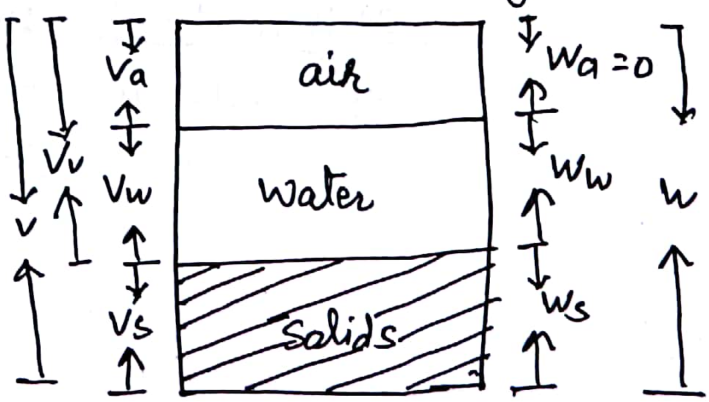


fig. Three phase diagram

- Although the soil is a three-phase system, it becomes two phase system in the following two conditions:-
  - (i) when soil is absolutly dry, then water phase disappears.
  - (ii) when soil is fully saturated, then there is no air phase.

- In a three phase diagram it is convention to write volume on the left side and mass on the right side.





V = total volume of soil.
V_a = volume of air.
V_w = volume of water
V_s = volume of solids
m = total mass of soil
m_a = mass of air = 0
m_w = mass of water
m_s = mass of solids.
W = total weight of soil.
W_a = ~~total~~ weight of air
W_s = weight of solids
W_w = weight of water

Volumetric Relationships:-

There are five volumetric Relationships used:-

① void-ratio(e): It is the ratio of the volume of the voids to the volume of solids.

$$e = \frac{V_v}{V_s} \quad \text{———— (1)}$$

② Porosity(n): It is the ratio of the volume of voids to the total volume.

$$n = \frac{V_v}{V} \quad \text{———— (2)}$$

Relation between vaid-ratio(e) and porosity(n)

$$\frac{1}{n} = \frac{V}{V_v} = \frac{V_v + V_s}{V_v} = \frac{V_v}{V_v} + \frac{V_s}{V_v} = 1 + \frac{V_s}{V_v} = 1 + \frac{1}{(V_v/V_s)} = \left(1 + \frac{1}{e}\right)$$

or, $\frac{1}{n} = \left(1 + \frac{1}{e}\right) = \frac{e+1}{e}$

or, $\frac{1}{n} = \frac{e+1}{e}$ or, $n = \frac{e}{(1+e)}$

or, $\boxed{n = \frac{e}{(1+e)}}$ ——— (3)

Now, we have

$\frac{1}{n} = \left(1 + \frac{1}{e}\right)$

or, $\frac{1}{e} = \frac{1}{n} - 1 = \left(\frac{1-n}{n}\right)$

or, $\boxed{e = \frac{n}{(1-n)}}$ ——— (4)

(3) Degree of Saturation (s) :-

It is ratio of the valume of water to the valume of vaids

$\therefore$ $\boxed{S = \frac{V_w}{V_v}}$ ——— (5)

(4) Percentage Air-vaids (n_a)

It is the ratio of the valume of air to the total volume.

$\boxed{n_a = \frac{V_a}{V}}$ ——— (6)

(5) Air-content (a_c)

It is the ratio of the valume of air to the valume of vaids.

$\boxed{a_c = \frac{V_a}{V_v}}$ ——— (7)

Inter-relationship between the percentage air voids (n_a) and air content (a_c) :-

$$n_a = \frac{V_a}{V} = \frac{V_a}{V_v} \times \frac{V_v}{V}$$

or, $n_a = \frac{V_a}{V_v} \times n$ [as $n = \frac{V_v}{V}$]

or, $n_a = a_c \times n$ [as $a_c = V_a/V_v$]

or, $\boxed{n_a = n a_c}$ ——— (8)

Volume-mass Relationships :-

- The volume mass relationships are in terms of mass-density.
- There are five different mass-densities used:-

① Bulk mass Density (ρ)

- It is defined as the total mass (m) per-unit total volume (V)

$$\rho = \frac{m}{V} \quad \text{——— (1)}$$

- also known as the "wet mass density" or "Bulk density" or "density".

② Dry mass density (ρ_d)

It is defined as the mass of solids per unit total volume.

$$\rho_d = \frac{m_s}{V} \quad \text{——— (2)}$$

- known as dry density.

③ Saturated mass density (ρ_{sat})

- It is the bulk mass density of a soil when it is fully saturated.

- $\rho_{sat} = \frac{M_{sat}}{V}$ ——— ③

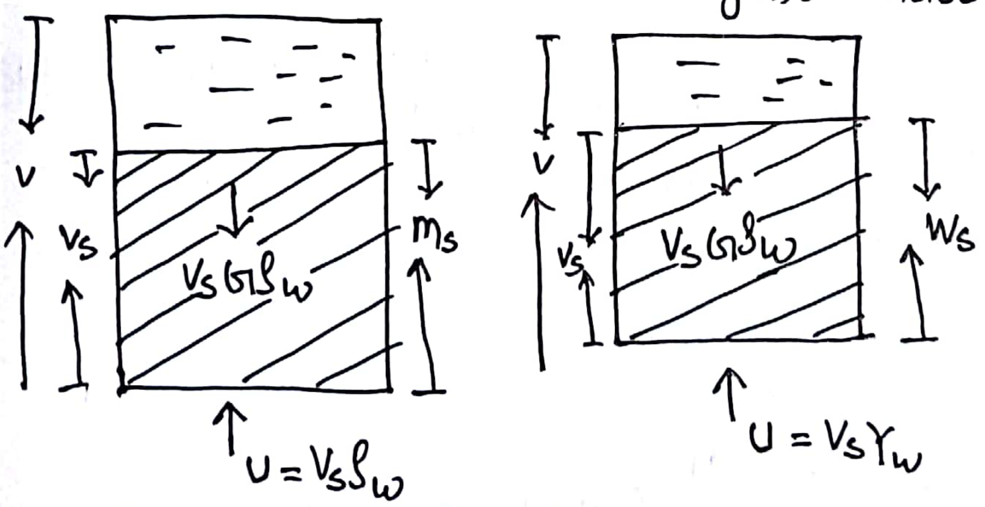
④ Submerged mass density (ρ')

When the soil exist below water, it is a submerged conditions. When a volume (v) of soil is submerged in water, it displace an equal volume of water.

· The submerged mass density (ρ') of the soil is defined as the submerged mass per unit of total volume.

$$\rho' = \frac{M_{sub}}{V} \quad \text{———} \quad ④$$

- also known as the buoyant mass density (ρ_b)



- Fig above shows a soil mass submerged under water. The soil solids which have a volume (V_s) are buoyed up by the water. The upthrust is equal to the mass of water displaced by the solids. Thus

$U = V_s \rho_w$ [as mass of water (M_w) $= \rho_w \cdot V_w = \rho_w \cdot V_s$ as $V_s = V_w$

Thus $M_{sub} = M_s - U = (V_s \cdot \rho_s) - U = (V_s \cdot \frac{\rho_s}{\rho_w} \cdot \rho_w) - V_s \rho_w$

or, $M_{sub} = (V_s \cdot G \cdot \rho_w) - V_s \rho_w$ [as $G = \frac{\rho_s}{\rho_w}$]

we have,-

$$\rho' = \frac{M_{sub}}{V} = \frac{(V_s \cdot G \cdot \rho_w) - (V_s \rho_w)}{V} \quad \left[\text{as } M_{usb} = (V_s \cdot G \cdot \rho_w) - (V_s \rho_w) \right]$$

or, $$\boxed{\rho' = \frac{V_s \rho_w (G-1)}{V}} \quad \text{——— (5)}$$

Alternatively:-

we can also consider the equilibrium of the entire valume (V). In this case, the total doconward mass, including the mass of water in the vaids is given by,:

$$M_{sat} = M_s + V_v \rho_w \quad \text{——— (6)}$$ [here valume of water = valume of valds as in submerged condition $V_v = V_w$]

the total upward thrust, including that on the water in vaids is given by

$$U = V\rho_w$$ [here V = valume of water + valume of salids]

Thus, the submerged mass is given by

$$M_{sub} = (M_s + V_v \rho_w) - V\rho_w \quad \text{——— (7)}$$

and as we know

$$\rho' = \frac{M_{sub}}{V}$$

or, $$\rho' = \frac{(M_s + V_v \rho_w) - V\rho_w}{V}$$

or, $$\rho' = \frac{M_{sat} - V\rho_w}{V} \quad [\text{as } M_{sat} = M_s + V_v \rho_w]$$

or, $$\rho' = \frac{M_{sat}}{V} - \frac{V\rho_w}{V}$$

or, $$\rho' = \frac{M_{sat}}{V} - \rho_w$$

or, $$\boxed{\rho' = \rho_{sat} - \rho_w} \quad \text{——— (8)}$$

The submerged density (ρ') is raughly one half of the saturated density.

⑤ Mass Density of solids:- (ρ_s)

It is the ratio of the mass of solids to the volume of solids.

$$\rho_s = \frac{M_s}{V_s} \qquad \text{———— (9)}$$

Volume - weight Relationship:-

- The volume-weight relationships are in terms of unit weights. The weight of soil per unit volume is known as unit weight.
- There are five different unit weights are used in computations.

① Bulk unit weight (γ):-

It is defined as the total weight per unit total volume. Thus

$$\gamma = \frac{W}{V} \qquad \text{———— (1)}$$

- also called total unit weight (γ_t) or wet unit weight.

② Dry unit-weight (γ_d)

- It is defined as the weight of solids per unit total volume. Thus

$$\gamma_d = \frac{W_s}{V} \qquad \text{———— (2)}$$

③ Saturated unit weight (γ_{sat})

- It is the bulk unit weight when the soil is fully saturated. Thus

$$\gamma_{sat} = \frac{W_{sat}}{V} \qquad \text{———— (3)}$$

④ Submerged unit weight (γ') :-

When the sail exists below water, it is a submerged conditions. A buoyant force acts on the sail salids. The submerged unit weight (γ') of the sail is defined as the submerged weight per unit of total valume. Thus

$$\gamma' = \frac{W_{sub}}{V} \text{ ——— } ④$$

- fig is similar to the previous one.
- The buoyant force (U) is equal to the weight of water displaced by the salids.

$$U = V_s \gamma_w \text{ ——— } ⑤$$

Thus

$$W_{sub} = W_s - U$$

$$W_{sub} = (V_s G \gamma_w - V_s \gamma_w) = V_s \gamma_w (G-1) \text{ ——— } ⑥$$

we know:-

$$\boxed{\gamma' = \frac{W_{sub}}{V} = \frac{V_s \gamma_w (G-1)}{V}}$$

Alternatively,:

we can also consider the equilibrium of the entire valume (V). The total downward force, including the weight of water in the vaids is given by.

$$W_{sat} = W_s + V_v \gamma_w \text{ ——— } ⑦$$

- The total upward farce, including that on the water in vaids is given by

$$U = V \gamma_w \text{ ——— } ⑧$$

Thus,- submerged unit weight is given by.-

$$W_{sub} = (W_s + V_v \gamma_w) - V\gamma_w \text{ ——— } ⑨$$

and we have,-

$$\gamma' = \frac{W_{sub}}{V}$$

substituting the value of (W_{sub}) we get:-

$$\gamma' = \frac{W_{sub}}{V} = \frac{(W_s + V_v\gamma_w) - V\gamma_w}{V}$$

or, $\gamma' = \frac{W_{sat} - V\gamma_w}{V}$ $\left[\text{as, } W_{sat} = W_s + V_v\gamma_w\right]$

or, $\gamma' = \frac{W_{sat}}{V} - \frac{V\gamma_w}{V}$

or, $\gamma' = \gamma_{sat} - \gamma_w$ ——— (10)

(5) Unit weight of soil solids :- (γ_s)

- It is equal to the ratio of the weight of solids to the volume of solids.

Thus;- $\gamma_s = \frac{W_s}{V_s}$ ——— (11)

Three phase diagram in terms of void-ratio

- The volume of solids is taken as unity in 3-Phase diagram.
- If the cross-sectional area of the soil is also taken unity, then the ~~volume~~ height of solids is unity.
- below ~~is~~ are the figures of three phase diagram in terms of void ratio.

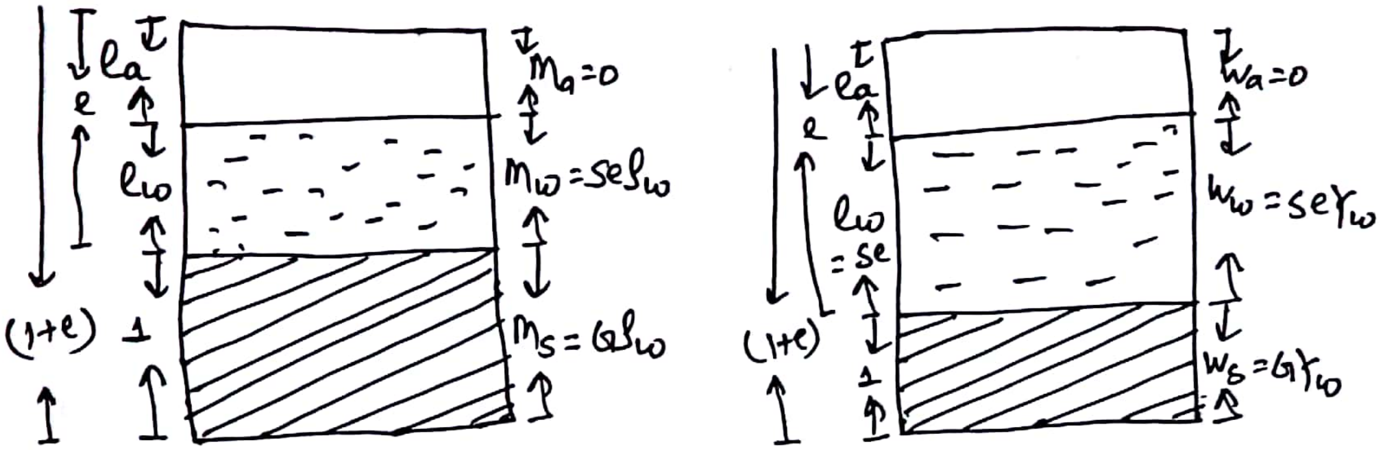


fig :. Three Phase diagram in terms of void-ratio

here,-

Void ratio(e) = $\frac{V_v}{V_s} = \frac{V_v}{1}$ [as $V_s = 1$]

or, $e = V_v$ or, $\boxed{V_v = e}$

- total volume (v) of soil is equal to the volume of voids(+) volume of solids thus,-

$\therefore$ $V = V_v + V_s = e + 1$

or, $\boxed{V = 1+e}$

- Volume of air is taken as (e_a)
- volume of water is taken as (e_w)
- Porosity (n) = $\frac{V_v}{V} = \frac{e}{(1+e)}$ [as, $V_v = e$; $V = 1+e$]
- Degree of saturation(s) = $\frac{V_w}{V_v} = \frac{e_w}{e}$

or, $S = \frac{e_w}{e}$ or, $\boxed{e_w = se}$

- volume of water = $e_w = se$

$\therefore$ volume of air = (volume of voids) − (volume of water)

$V_a = V_v - V_w$

or, $e_a = e - se$ [as, $V_a = e_a$; $V_v = e$; $V_w = se$]

or, $\boxed{e_a = e(1-s)}$

- percentage air voids $(n_a) = \frac{V_a}{V} = \frac{e(1-s)}{(1+e)}$ $\left[\begin{array}{l} a, V_a = e_a = e(1-s) \\ V = (1+e) \end{array} \right]$
- air content $(a_c) = \frac{V_a}{V_v} = \frac{e(1-s)}{e} = (1-s)$

Mass Densities can be expressed in terms of the void-ratio :-

- Percentage air voids $(n_a) = \frac{V_a}{V} = \frac{e(1-s)}{(1+e)}$ $\left[\begin{array}{l} as,\ V_a = e_a = e(1-s) \\ V = (1+e) \end{array}\right]$
- air content $(a_c) = \frac{V_a}{V_v} = \frac{e(1-s)}{e} = (1-s)$

Mass Densities can be expressed in terms of the void-ratio :-

- Bulk mass density $(\rho) = \frac{M}{V} = \frac{M_s + M_w}{(1+e)}$

or, $\rho = \frac{G\rho_w + V_w\rho_w}{(1+e)}$

or, $\rho = \frac{G\rho_w + V_w\rho_w}{(1+e)}$

or, $\rho = \frac{G\rho_w + e_w\rho_w}{(1+e)}$ $[as,\ V_w = e_w]$

or, $\rho = \frac{G\rho_w + se\rho_w}{(1+e)}$ $[as,\ e_w = se]$

or, $\boxed{\rho = \frac{\rho_w[G + se]}{(1+e)}}$

[here, $M_s = G\rho_w$ as $G = \frac{\rho_s}{\rho_w}$ — (i)
and $\rho_s = \frac{M_s}{V_s}$ or, $\rho_s = M_s$ [as $V_s = 1$]
$\therefore \rho_s = M_s$ ——— (ii)
Now, $G = \frac{\rho_s}{\rho_w}$ or, $G = \frac{M_s}{\rho_w}$ or, $M_s = G\rho_w$]

- Dry mass density :- $(\rho_d) = \frac{M_s}{V} = \frac{G\rho_w}{(1+e)}$ $\left[\begin{array}{l} as,\ M_s = G\rho_w \\ V = 1+e \end{array}\right]$

- Saturated mass density :- $(\rho_{sat}) = \frac{M_{sat}}{V}$

It is the bulk mass density of the soil when it is fully saturated.
When soil is fully saturated then, the degree of saturation is 1.0 (i.e 100%) so $S=1$.

we have,-

bulk mass density $(\rho) = \frac{\rho_w[G+se]}{(1+e)}$

Substituting $s=1$; for the fully saturated soil.

so, $\rho_{sat} = \frac{\rho_w[G+(1\times e)]}{(1+e)}$ when $s=1$.

or, $\boxed{\rho_{sat} = \frac{\rho_w[G+e]}{(1+e)}}$

- Submerged mass density $(\rho') = \rho_{sat} - \rho_w$

or, $\rho' = \frac{(G+e)\rho_w}{(1+e)} - \rho_w$

or, $\rho' = \frac{(G+e)\rho_w - \rho_w(1+e)}{(1+e)} = \frac{G\rho_w + e\rho_w - \rho_w - e\rho_w}{(1+e)}$

$= \frac{G\rho_w - \rho_w}{(1+e)} = \frac{\rho_w(G-1)}{(1+e)}$

or, $\boxed{\rho' = \frac{\rho_w(G-1)}{1+e}}$

In the case the soil is not fully saturated, the submerged mass-density is given by

$\rho' = \rho - \rho_w$

or, $\rho' = \frac{(G+se)\rho_w}{(1+e)} - \rho_w$

or, $\rho' = \frac{(G+se)\rho_w - \rho_w(1+e)}{(1+e)} = \frac{[(G+se)-(1+e)]\rho_w}{(1+e)}$

or, $\rho' = \frac{[G+se-1-e]\rho_w}{(1+e)}$ or, $\rho' = \frac{[G-1+es-e]\rho_w}{(1+e)}$

or $\rho' = \frac{[G-1-e+es]\rho_w}{(1+e)}$

or $\rho' = \frac{[(G-1)-e(1-s)]\rho_w}{(1+e)}$

Now, when the soil is fully saturated $s=1$,
By substituting $s=1$ in the eqn.:

$\rho' = \frac{[(G-1)-e(1-s)]\rho_w}{(1+e)}$ we get

or, $\rho' = \frac{[(G-1)-e(1-1)]\rho_w}{(1+e)}$

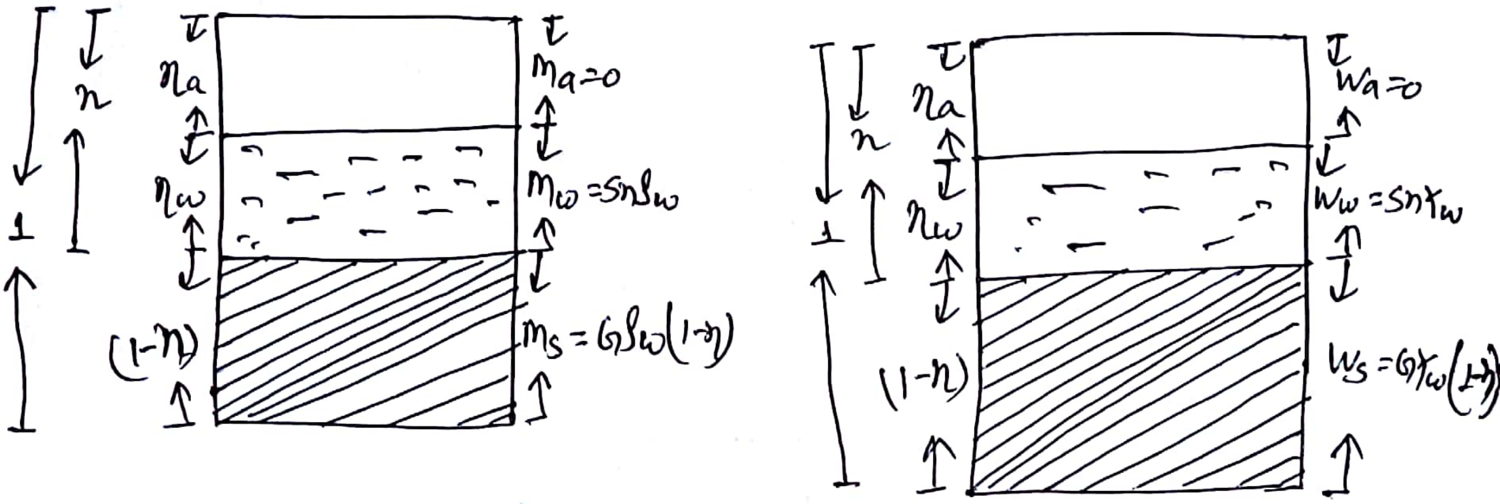
or, $\rho' = \frac{[(G-1)-0]\rho_w}{(1+e)} = \frac{[G-1]\rho_w}{1+e}$.

or, $\rho' = \frac{[G-1]\rho_w}{(1+e)}$ which is same as we obtain in the previous case.

Equations in weight units (or weight equations) can be expressed in terms of void-ratio:→

- Bulk unit weight $(\gamma) = \frac{(G+se)\gamma_w}{(1+e)}$
- Dry unit weight $(\gamma_d) = \frac{G\gamma_w}{(1+e)}$
- Saturated unit weight $(\gamma_{sat}) = \frac{(G+e)\gamma_w}{(1+e)}$
- Submerged unit weight $(\gamma') = \frac{(G-1)}{(1+e)}\gamma_w$
  [when soil is fully saturated then $s=1$ (i.e. 100%)]

Three- Phase diagram in terms of porosity



Three phase diagram in terms of porosity

here,-
Volume of air (V_a) is represented by (n_a)
Volume of water (V_w) is represented by (n_w)
total Volume of soil is taken as unity.

- By definition,-

$$\text{Porosity } (n) = \frac{V_v}{V} = \frac{V_v}{1} \quad [\text{as total volume of soil} = 1]$$

$$\text{or, } n = \frac{V_v}{1} \quad \text{or, } V_v = n$$

Thus, $\boxed{V_v = n}$

- Volume of solids is thus $(1-n)$ [i.e. V_s = total volume - Volume of voids.]

$$V_s = 1 - V_v = 1 - n$$

$$\therefore \boxed{V_s = 1 - n}$$

- Void- ratio, $e = \frac{V_v}{V_s} = \frac{n}{(1-n)}$ or, $\boxed{e = \frac{n}{1-n}}$

- Bulk mass density (ρ) $= \frac{m}{V} = \frac{m_s + V_w \rho_w}{V}$

or, $\rho = \frac{V_s \rho_s + V_w \rho_w}{V}$

or, $\rho = \frac{\left[(1-n)\frac{\rho_s}{\rho_w} \times \rho_w\right] + [V_w \rho_w]}{1}$ as, $V = 1$

or $\rho = \frac{\left[(1-n)\, G\rho_w\right] + Sn\rho_w}{1}$ $\left[\text{as, } S = \frac{V_w}{V_v} \text{ or, } S = \frac{V_w}{n} \text{ or, } V_w = Sn\right]$

or, $\boxed{\rho = [G(1-n) + Sn]\rho_w}$

- Dry mass density (ρ_d) $= \frac{m_s}{V} = \frac{G\rho_w(1-n)}{1}$

or, $\boxed{\rho_d = G\rho_w(1-n)}$

- Saturated mass-density (ρ_{sat})

$\rho_{sat} = \frac{M_{sat}}{V} = \frac{m_s + m_w}{1}$

or, $\rho_{sat} = \frac{G\rho_w(1-n) + Sn\rho_w}{1}$ $\left[\text{as, } m_s = G\rho_w(1-n) \text{ and } m_w = Sn\rho_w\right]$

or, $\rho_{sat} = \left[\frac{G(1-n)\rho_w + 1 \times n\rho_w}{1}\right]$ $\left[\text{as, at saturated cond}^n\ S = 1\right]$

or, $\rho_{sat} = [G(1-n) + n]\rho_w$

- Submerged mass density (ρ')

$\therefore \rho' = \frac{M_{sub}}{V} = \frac{(V_s G\rho_w) - (V_s\rho_w)}{V}$ $\left[\frac{\text{downward mass} - \text{upward mass}}{\text{total volume}}\right]$

or, $\rho' = \frac{V_s\rho_w(G-1)}{V}$

thus:-

Submerged mass-density

$$(\rho') = \frac{V_s \rho_w (G-1)}{V} = \frac{(1-n)\rho_w (G-1)}{1} \quad [\text{as } V_s = (1-n)]$$

or, $\boxed{\rho' = (G-1)(1-n)\rho_w}$

<u>Note</u>:- The above equactions in terms of porosity can also be derived directly by substituting $e = \frac{n}{(1-n)}$ in the equaction developed for void-ratio terms.

<u>The equactions in terms of weight units:-</u>

- The above equactions can be written in terms of unit weight as below:-

- Bulk unit weight $(\gamma) = [G(1-n)+Sn]\gamma_w$
- Dry unit weight $(\gamma_d) = G\gamma_w(1-n)$
- Saturated unit weight $(\gamma_{sat}) = [G(1-n)+n]\gamma_w$
- Submerged unit weight $(\gamma') = (G-1)(1-n)\gamma_w$

Relationship between the void-ratio and the water content:-

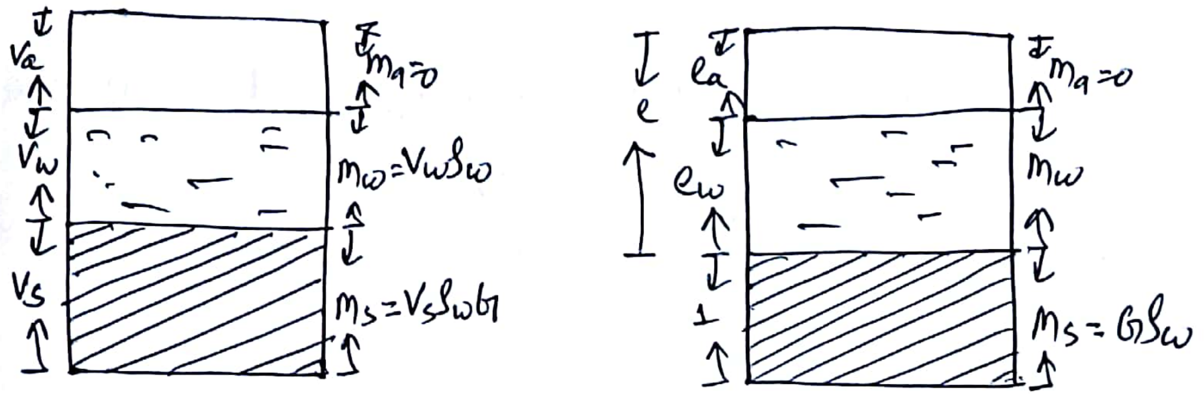


fig:- The three phase diagram

- The water content is defined as the ratio of mass of water to mass of solids.

$$w = \frac{m_w}{m_s}$$ where, w = water content

m_w = mass of water

m_s = mass of solids.

or, $$w = \frac{V_w \rho_w}{V_s \rho_s} \quad \text{———} \; (1)$$

we know that:

- degree of saturation (S) = $\dfrac{\text{Volume of water } (V_w)}{\text{Volume of voids } (V_v)}$ ——— (2)

- Specific gravity (G) of solid particles = $\dfrac{\text{density of solid } (\rho_s)}{\text{density of water } (\rho_w)}$

$$\text{or, } \rho_s = G\rho_w \quad \text{———} \; (3)$$

from eqn (2)

$$S = \frac{V_w}{V_v}$$

$$\text{or, } V_w = SV_v \quad \text{———} \; (4)$$

from eqⁿ ①

$$w = \frac{V_w \rho_w}{V_s \rho_s}$$

Substituting the value from eqⁿ ③ and eqⁿ ④ we get,-

$$W = \frac{S V_v \rho_w}{V_s G \rho_w} \quad \left[\begin{array}{l} \text{as } V_w = S V_v \\ \rho_s = G \rho_w \end{array} \right]$$

or, $$W = \frac{S V_v}{V_s G}.$$

But $\frac{V_v}{V_s}$ = void-ratio (e). Thus,-

$$\left. \begin{array}{l} W = \frac{Se}{G} \\ \text{or, } e = \frac{WG}{S} \end{array} \right\} \text{———} ⑤$$

For a fully saturated soil $S=1$

Putting the value of $S=1$ in eqⁿ ⑤ we get,-

$$W = \frac{Se}{G} = \frac{1 \times e}{G} = \frac{e}{G}$$

$$\left. \begin{array}{l} \text{or, } w = \frac{e}{G} \\ \text{or, } e = wG \end{array} \right\} \text{———} ⑥$$

Alternatively eqⁿ ⑤ and ⑥ can be derived using the 3-Phase diagram in terms of void-ratio.

we have,-

$$W = \frac{m_w}{m_s} = \frac{S e \rho_w}{G \rho_w}$$

$\Big[$ as $m_w = e_w \rho_w$

$= S e \rho_w \left[\text{as } S = \frac{V_w}{V_v} = \frac{e_w}{e} \text{ or, } e_w = Se \right] \Big]$

And,-

$m_s = V_s \rho_s = 1 \times \rho_s = \frac{\rho_s}{\rho_w} \times \rho_w = G \rho_w$.

or, $W = \dfrac{se\rho_w}{G\rho_w} = \dfrac{se}{G}$

or, $w = \dfrac{se}{G}$

$e = \dfrac{wG}{s}$ — the same eqn as eqn (5)

for fully saturated soil $s=1$.

Putting the value of $s=1$ in eqn (5) above, we gets:-

$W = \dfrac{se}{G} = \dfrac{1\times e}{G} = \dfrac{e}{G}$

or, $w = \dfrac{e}{G}$

$e = wG$ → the same eqn as eqn (6)

Expressions for mass density in terms of water content

- The expression for mass density can be written in terms of water content by writting the void-ratio in terms of water-content.

We have,-

- Bulk mass density (ρ)

$\rho = \dfrac{(G+se)\rho_w}{(1+e)}$ [from void ratio terms]

or, $\rho = \dfrac{(G+wG)\rho_w}{1+\left(\frac{wG}{s}\right)} = \dfrac{(1+w)G\rho_w}{1+\left(\frac{wG}{s}\right)}$ $\left[\text{as, } w = \dfrac{se}{G} \text{ or } wG = se,\ w = \dfrac{se}{G} \text{ or } e = \dfrac{wG}{s}\right]$

or, $\rho = \dfrac{(1+w)G\rho_w}{1+(wG/s)}$ ———— (1)

- If the soil is fully saturated, then $S=1$, then the eq^n ① becomes

$$\rho = \frac{(1+w)G\rho_w}{1+(wG/S)}$$

Putting $S=1$, we get,-

$$\rho \text{ or } \rho_{sat} = \frac{(1+w)G\rho_w}{(1+wG)} \quad \text{———} \quad (2)$$

- Submerged density is given by-

$$\rho_{sub} = \rho_{sat} - \rho_w = \frac{(1+w)G\rho_w}{(1+wG)} - \rho_w$$

$$\text{or, } \rho_{sub} = \frac{G\rho_w + wG\rho_w - \rho_w(1+wG)}{(1+wG)}$$

$$\text{or, } \rho_{sub} = \frac{G\rho_w + \underline{wG\rho_w} - \rho_w - \cancel{\rho_w}\, wG\rho_w}{(1+wG)}$$

$$\text{or, } \rho_{sub} = \frac{G\rho_w - \rho_w}{(1+wG)}$$

$$\text{or, } \rho_{sub} = \frac{\rho_w(G-1)}{(1+wG)} \quad \text{———} \quad (3)$$

- Dry mass density $(\rho_d) = \dfrac{G\rho_w}{(1+e)}$

$$\text{or, } \rho_d = \frac{G\rho_w}{1+\left(\frac{wG}{S}\right)} \quad \text{———} \quad (4) \text{ as, } e = \frac{wG}{S}$$

Now, from eqⁿ ① we have,-

$$\rho = \frac{(1+w)\,G\rho_w}{1+\left(\frac{wG}{S}\right)}$$

or, $$1+\left(\frac{wG}{S}\right) = \frac{(1+w)\,G\rho_w}{\rho} \quad \text{——— (5)}$$

from eqⁿ ④ we have,-

$$\rho_d = \frac{G\rho_w}{1+\left(\frac{wG}{S}\right)}$$

or, $$1+\left(\frac{wG}{S}\right) = \frac{G\rho_w}{\rho_d} \quad \text{——— (6)}$$

Equating eqⁿ ⑤ and ⑥ we have,-

$$\frac{(1+w)\,G\rho_w}{\rho} = \frac{G\rho_w}{\rho_d}$$

[equating the right hand side of eqⁿ ⑤ and ⑥ because left hand sides are equal.]

or, $$\rho_d = \frac{\rho}{(1+w)} \quad \text{——— (7)}$$

- from eqⁿ ④ we have,-

$$\rho_d = \frac{G\rho_w}{1+\left(\frac{wG}{S}\right)}$$

For a given water-content (w), a soil becomes saturated when $S=1$: The dry density of soil in such a condition can be represented as:

$$(\rho_d)_{sat} = \frac{G\rho_w}{(1+wG)} \quad \text{——— (8)}$$

where $(\rho_d)_{sat}$ is called saturated dry density.

Difference betn $(\rho)_{sat}$ and $(\rho_d)_{sat}$

In the ρ_{sat}, the water content of a partially saturated is increased so that all the voids are filled with water.

In the $(\rho_d)_{sat}$; the water content is kept constant and the air voids are removed by compaction so that all the remaining voids are saturated with water.

Equactions in terms of weight units:-

The above equactions can be rewritten in terms of unit weights as under.

$$-\gamma = \frac{(1+w)\,G\gamma_w}{1+\left(\frac{wG}{S}\right)} \qquad \text{——— (9)}$$

$$\gamma_{sat} = \frac{(1+w)\,G\gamma_w}{(1+wG)} \qquad \text{——— (10)}$$

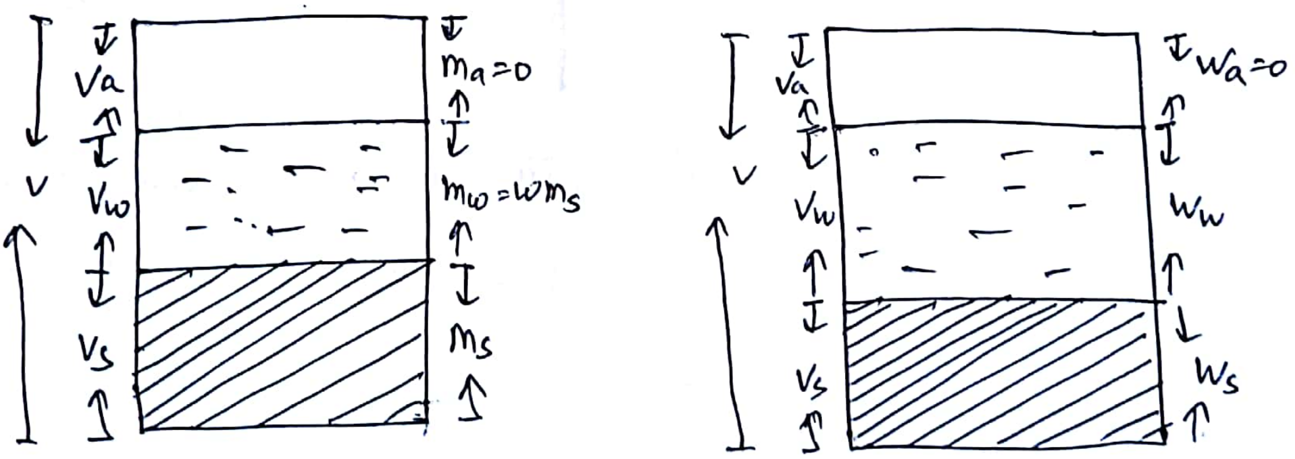
$$\gamma_{sub} = \frac{(G-1)\,\gamma_w}{(1+wG)} \qquad \text{——— (11)}$$

$$\gamma_d = \frac{G\gamma_w}{1+\left(\frac{wG}{S}\right)} \qquad \text{——— (12)}$$

$$\gamma_d = \frac{\gamma}{(1+w)} \quad \text{——— (13)}$$

$$(\gamma_d)_{sat} = \frac{G\gamma_w}{(1+wG)} \quad \text{——— (14)}$$

Relationship between Dry mass density and percentage Air voids:-



Three-Phase diagram:.

Now:

$$V = V_s + V_w + V_a$$

$$\text{or, } 1 = \frac{V_s}{V} + \frac{V_w}{V} + \frac{V_a}{V}$$

But. $\frac{V_a}{V} = n_a$ (Percentage air voids)

Thus:

$$1 = \frac{V_s}{V} + \frac{V_w}{V} + n_a$$

$$\text{or, } (1-n_a) = \frac{V_s}{V} + \frac{V_w}{V}$$

$$\text{or, } (1-n_a) = \frac{(M_s/G\rho_w)}{V} + \frac{M_w/\rho_w}{V} \left[\text{as, } V_s = \frac{M_s}{\rho_s} = \frac{M_s}{\frac{\rho_s}{\rho_w}\times\rho_w} = \frac{M_s}{G\rho_w}\right]$$

or, $(1-n_a) = \frac{\rho_d}{G\rho_w} + \frac{(wm_s)/\rho_w}{V}$ $\left[\text{as. } \rho_d = \frac{m_s}{V} \text{ and } w = \frac{m_w}{m_s} \text{ or, } m_w = wm_s\right]$

or, $(1-n_a) = \frac{\rho_d}{G\rho_w} + \frac{w\rho_d}{\rho_w}$ $\left[\text{as, } \rho_d = \frac{m_s}{V}\right]$

or, $(1-n_a) = \frac{\rho_d}{\rho_w}\left(\frac{1}{G} + w\right)$

or, $(1-n_a) = \left(w + \frac{1}{G}\right)\frac{\rho_d}{\rho_w}$

or, $$\boxed{\rho_d = \frac{(1-n_a)\,G\rho_w}{1+wG}} \quad \text{———— (1)}$$

When soil is fully saturated at a given water content, then $n_a = 0$; then the eqn 1 can be written as.

$$(\rho_d)_{n_a=0} = \frac{G\rho_w}{(1+wG)}$$

or

$(\rho_d)_{sat}$

- In terms of unit weight eqn (1) can be written as:-

$$\gamma_d = \frac{(1-n_a)\,G\gamma_w}{(1+wG)} \quad \text{———— (2)}$$